dYdX Short Tokens & Leveraged Long Tokens

dYdX

April 25, 2018

Abstract

We present an extension of the dYdX Margin Trading Protocol that allows tokenization of short and leveraged long positions. Short Tokens and Leveraged Long Tokens implement the standard ERC20 interface, making them usable with existing smart contracts, trading platforms, and wallets. We also show how the price of each token can easily be calculated from the price of the *base token* used in the positions. Lastly, we prove the robustness of this price, even if users are unwilling to interact directly with the Margin Trading Protocol.

1 Introduction

With dYdX it is possible to create a token that has a price negatively correlated to an underlying crypto asset. We refer to this type of a token as a “short token”. The price of the token increases when the price of the underlying decreases.

Similarly, it is possible to create a token that has a price that moves at a multiple of the price of an underlying crypto asset. We refer to this type of a token as a “leveraged long token”. The price of the token increases by a factor greater than one when the price of the underlying increases.

We refer to short and leveraged long tokens together as “margin tokens”.

Margin tokens are backed by dYdX margin positions, and therefore have defined prices at which they should trade. These tokens allow traders to get short and leveraged long exposure by simply buying a token, with no need to borrow or sell the underlying asset.

2 Description

A dYdX margin token is a freely tradable ownership interest in a dYdX margin position based upon the ERC20 standard. Each type of margin token has a specified interest rate, expiration date, and amount of *held token* locked in the position per unit *owed token* sold through the dYdX margin position. Each token is fungible, transferable, and can be traded in any amount.

2.1 Creation

Anyone can create a new margin token. Each margin token is its own smart contract. When a margin token is created, it represents an ownership interest in a margin position on the base dYdX Margin Trading Protocol[1]. Each type of margin token represents a different set of terms including expiration date, interest rate, and amount of *held token* locked in the position per unit *owed token* of principal.

Token holders enter into these positions independently of one another. There is no central manager of the position and each holder is responsible for managing their position independently of others.

2.2 Minting

Once a margin token is created, anyone can add to the supply of the token (mint) by increasing the margin position backing the token. This is done by borrowing and selling *owed tokens* and putting up either *held token* or *owed token* as margin deposit exactly as per opening a position in the dYdX Margin Trading Protocol[2]. The newly minted supply of margin tokens conform to exactly the same specs as the original margin token (e.g. borrowed from lender at same interest rate and expiration date).

2.3 Closing

Margin token holders can close their position at any time (independent of other token holders) through the normal dYdX closing procedure[3]. Closing will cause the received *held token* or *owed token* to be sent to the margin token holder, and the holder's margin tokens to be burned.

Since margin tokens are backed by positions, they have a set maximum duration and can also be margin called if the price has moved against the position[4]. In these cases, token holders can either: sell their tokens, close their tokens, or wait for an auction closing mechanism to automatically convert their tokens into *held token*. This closing mechanism is the same Dutch Auction contract described in the dYdX Protocol Whitepaper[5].

---

[1] dYdX Whitepaper: 3.1 Margin Trading
[2] dYdX Whitepaper: 3.1.4.4 Position Opening
[3] dYdX Whitepaper: 3.1.4.5 Closing
[4] dYdX Whitepaper: 3.1.4.6 Calling
[5] dYdX Whitepaper: 3.1.4.6 Calling

3 Implementation

A powerful concept enabled by dYdX is that positions can be extended by smart contracts. This enables complex behavior external to the base protocol.

One example of this is that margin positions can be extended by a smart contract which then creates tokens (known as the *ERC20Position* contract). Each token grants ownership of a portion of the position to its holder.

3.1 Creation

dYdX allows ownership of positions to be transferred to new addresses. Anyone can use this to create a new margin token by assigning ownership of a position to a new *ERC20Position* smart contract. The *ERC20Position* contract will then mint tokens proportional to the size of the position it received, and assign them to the creator. Tokens are freely tradable as soon as they are minted.

3.2 Minting

dYdX positions can be increased in much the same way that they are opened. A trader uses a loan offering and a buy order to borrow and sell *owed token* in exchange for *held token*. In order to increase an existing position, the loan offering must share many of the same properties with the existing position, such as the specific underlying tokens, the interest rate, and the maximum duration.

When lenders enter into the position later, they are already owed the interest that has previously accrued in the position. This means they should lend more *owed token* than the amount of principal that they are awarded in order to be on-par with other existing lenders. Because interest is compounded continuously, the amount they lend is equal to $P \times e^{rt}$.[6]

When value is added, the ratio of *held token* stored in the position per unit *owed token* of principal must remain the same. Therefore, based on the *held token* received from trading away the lender's *owed token*, the deposit from the trader is calculated such that the ratio is maintained.

Anyone can use this process to mint new margin tokens on an existing *ERC20Position*. When value is added to a margin position that is tokenized, *Margin* calls the corresponding *ERC20Position* contract to tell it additional value has been added. When this happens, *ERC20Position* mints new tokens proportional to the amount that was added and grants them to the transaction initiator (trader).

3.3 Closing

dYdX positions can be closed in increments. This allows token holders to close their portion of the underlying margin position independent to other holders.

[6] Compound Interest on Wikipedia: Continuous Compounding

A token holder can close by sending a transaction directly to the *Margin* smart contract. The following steps then happen atomically:

1. The *Margin* smart contract asks *ERC20Position* contract if it is ok for the transaction initiator to close some portion of the position
2. If the transaction initiator does in fact own at least as many tokens as the requested close amount, the token contract burns the close amount of tokens and tells the *Margin* smart contract that it is ok for the transaction initiator to close that amount
3. *Margin* marks the amount of the position as closed
4. The transaction initiator is paid out directly

4 Value Analysis

Because margin tokens are backed by positions, they have a defined price at which they should trade. The lower-bound of the price is the payout from closing a portion of the underlying position. The upper-bound of the price is the cost of minting new tokens. We will show that the payout from closing and the cost to mint are exactly equal.

Therefore, there is an exact price in *quote token*[7] for each margin token given the following parameters: price of the *base token*[8], interest rate, time since creation, and amount of collateral per unit principal. The price is maintained by economic incentives that exist in the form of arbitrage opportunities if the price deviates from this value.

4.1 Position Value

4.1.1 Payout of a Position

The payout of a margin position is defined by:

1) The price of the *held token* in the position that the owner is entitled to.
2) Minus the price of the *owed token* that must be repaid to the lender.

We define the price of the *held token* in the position to be the price of a single *held token* (p_H) times the number of *held tokens* held as collateral in the position (C). Then the total price of all the *held tokens* in the position is[9]:

$$p_H \times C$$

We define the price of the *owed token* that must be repaid to the lender to be equal to the price of a single *owed token* (p_O) times the number of *owed tokens* that must be repaid. The number of *owed tokens* that must be repaid is continuously compounded on the principal amount of *owed tokens* originally lent by the lender (P). As explained above, the equation for continuously compounded interest is $P \times e^{rt}$. We will denote the value of e^{rt} at the time of closing to be I. Then the total price of all the *owed tokens* to be repaid is:

$$p_O \times P \times I$$

Therefore the payout from closing the position is:

[7] dYdX Whitepaper: 3.1.3 Overview. *quote token* is the token in which the trade is priced- e.g. what you are shorting against for short selling

[8] dYdX Whitepaper: 3.1.3 Overview. *base token* is the token being traded - e.g. what you are shorting for short selling

[9] We assume no slippage throughout these calculations

$$\textit{position payout} \quad = \quad (p_O \times C) - (p_O \times P \times I)$$

4.1.2 Cost of Increasing a Position

In order to mint new tokens for an existing position, a trader must increase the size of the margin position. The dYdX Margin Trading Protocol enforces that the ratio between C and P remains constant. New lenders must lend $e^{rt} = I$ times as many *owed tokens* in order to be granted a commensurate share in the loan.

Therefore, in order to increase the position by some multiple, X, a trader will have to lock up $X \times C$ *held tokens* and will borrow $X \times P \times I$ *owed tokens* from the lender. The total price that the trader will have to pay as margin deposit in order to increase the position by the multiple X is the difference between the locked-up value and the lent value:

$$(p_H \times C \times X) - (p_O \times P \times I \times X)$$

$$= \quad X \times (p_H \times C - p_O \times P \times I)$$

$$= \quad X \times (\textit{position payout})$$

Thus, the cost to a trader to increase a position by some fraction is proportional to the payout of that position. It follows that to mint any *margin token* the value paid is proportional to the payout of the existing tokens. This also assumes there is lending liquidity to lend tokens at the same terms of the position. If there is no lending liquidity, increasing a position is not possible.

4.2 Margin Token Value

4.2.1 Price Stability of Margin Tokens

We now consider the payout per each individual margin token. To get the payout of a single margin token, we take the payout of the entire position and divide by the total supply of tokens (M).

$$\textit{payout per margin token} \quad = \quad \frac{\textit{position payout}}{M}$$

Additionally, if we are to mint new margin tokens in an amount, N, then using our equation from 4.1.2, X would be defined as N divided by M. Therefore:

$$\textit{cost to mint N margin tokens} \quad = \quad X \times (\textit{position payout}) \quad = \quad N \times \frac{\textit{position payout}}{M}$$

$$\textit{minting cost per margin token} \quad = \quad \frac{\textit{position payout}}{M}$$

The minting cost per margin token is exactly equal to the payout per margin token. This property provides strong economic incentives to keep the market price of any margin token at this value.

If the market price of a margin token were lower than the payout of the token then an arbitrage opportunity would exist; there would exist an economic incentive to buy the tokens at the market price and receive the payout from closing the position using these tokens. Therefore:

$$\textit{payout} \quad \leq \quad \textit{market price}$$

Likewise, if the market price of a margin token is higher than the cost to mint a token, then a different arbitrage opportunity would be created; there would exist an economic incentive to mint new tokens and sell them on the open market. Therefore:

$$\textit{market price} \quad \leq \quad \textit{minting cost}$$

We have already shown that on a per-token basis, the payout is equal to the minting cost. Because the market price of any margin token is bounded by these values, then the market price is equal to:

$$p_{margin\ token} \quad = \quad \frac{(position\ payout)}{M} \quad = \quad (p_H \times C \, - \, p_O \times P \times I) \, / \, M$$

In later sections, we will show how this equation can be simplified by choosing M carefully.

Even if most token holders do not want to open or close margin positions directly, they can rely on economic incentives to keep the market price of the margin token close to its fair value. That is, anyone can enter into a margin position without needing to ever interact with the base dYdX Margin Trading Protocol.

It is important to note that there may be some fees for closing or increasing a position. These could include buy/sell fees, loan relayer fees, and gas costs. Therefore, we expect the token price to be stable within a small spread dictated by these fees.

In addition, the upper bound of the price breaks down whenever lending liquidity dries up; there is no way to mint new tokens without loan offerings. However, the lower bound will always hold.

4.2.2 Short Token Price

In short selling the *held token* is the *quote token*, the denomination in which assets are typically priced. Therefore $p_H = 1$. The *owed token* is the *base token* and is the asset being sold short, so we define $p_B = p_O$.

This makes the price per short token equal to:

$$p_{(Short\ Token)} \quad = \quad (C \, - \, p_B \times P \times I) \, / \, M$$

For *short tokens*, we set the number of *margin tokens*, M, be equal to the principal, P. Then the price of a *short token* is:

$$p_{(Short\ Token)} = \frac{C}{P} - p_B \times I$$

Since the ratio of C to P remains constant for a position (even if increased or partially-closed), we will represent this ratio as Q. This yields:

$$\boxed{p_{(Short\ Token)} = Q - (I \times p_B)}$$

Therefore, the price of a *short token* is negatively correlated with the price of the *base token*. The ratio is -1:1 when I is close to 1, which is the case for recently opened positions and/or low interest rate loans. However, I is monotonically-increasing, and as it increases, the payout of the *short token* becomes more volatile with respect to the *base token*. In addition, the maximum theoretical payout of the token is Q, occurring if the price of a *base token* is zero. Therefore, a lower collateral ratio decreases the value of each token without decreasing the volatility with respect to the *base token*, allowing for a higher leverage ratio.

4.2.3 Leveraged Long Token Price

For leveraged long trading the *owed token* is the *quote token*, the denomination in which assets are typically priced. Therefore $p_O = 1$. The *held token* is the *base token* and is the asset being bought with leverage, so we define $p_B = p_H$.

This makes the price per leveraged long token equal to:

$$p_{(Leveraged\ Long\ Token)} = (p_B \times C - P \times I) / M$$

For leveraged long tokens, we set the number of margin tokens, M, be equal to the collateral amount in *held token*, C. Then the price of a *leveraged long token* is:

$$p_{(Leveraged\ Long\ Token)} = p_B - \frac{P}{C} \times I$$

As before, representing the constant ratio of C to P as Q, yields:

$$\boxed{p_{(Leveraged\ Long\ Token)} = p_B - (I / Q)}$$

Therefore, the payout of an individual leveraged long token changes by the same amount as the price of an individual *base token*. However because of the negative term, the payout is less than the price of a *base* *token*. At a constant point in time, the negative term is governed by Q, which is determined by the ratio between collateral and principal in the position. Therefore, a lower collateral ratio decreases the value of each token without decreasing the volatility with respect to the *base token*, allowing for a higher leverage ratio.

5 Summary

dYdX Short and Leveraged Long Tokens give traders an easy way to get short and leveraged exposure to underlying crypto assets. They implement the ERC20 standard, and can be easily integrated into existing exchanges, wallets, and dApps.

The tokens use decentralized mechanisms to achieve price stability: due to the economic incentives to mint margin tokens (by increasing a position) or burn margin tokens (by closing a position), the price of these margin tokens should remain extremely close to their fair value even if the vast majority of token holders do not close or mint tokens. This allows for more advanced financial strategies without added complexity for exchanges or end users.